Average or Measures of Central Tendency

Definition : An average is a single value within The range of values (or data) which represents The whole values (or data).

Why it's called measure of Central tendency

In any frequency distribution (you got in your high school syllabus) table, The tabulated values show small frequencies at The beginning and end of The table. While in The middle part frequency is highest. This shows That near The central part of The table (or distribution) most of The values get gather. Such values are known as measure of Central Tendency or Average.

So, average of any distribution is a single value and it is a basis of comparision with other values of The group or series.

The characterisitics of an average

1. It should be easy to understand.
2. It should be easily calculated and while doing calculation all values of The group must come into consideration.
3. It must not be affected by extreme values.
4. It should be rigidly defined and must be capable for using further calculations.

There are five types of averages

(1) Arithmetic mean (A.M) (2) Geometric mean (G.M.)
(3) Harmonic mean (H.M) (4) Median (5) Mode

All Three means (A.M, G.M., H.M.) indicate the central value of any distribution, but Median and Mode indicate the value in the central position of any distribution.

So, Mean is mathematical average and Median and Mode are Average of Position

Note: If it is not specifically mentioned, we understand mean means Arithmetic mean (A.M.).

Definition of A.M.

Simple A.M. If we have $x_1, x_2, \cdots, x_n$ values, Then their ~~their~~ A.M is defined as

$$\bar{x}\ (A.M.) = \frac{\text{Some of values}}{\text{No. of value}} = \frac{\Sigma x}{n}$$

Weighted A.M. When x_1 has frequencies f_1 times, x_2 has f_2 times, ------ x_n has f_n times Then we define their A.M as

$$\bar{x} = \frac{\Sigma\,(\text{each value}) \times (\text{respective frequency})}{\text{Total frequencies}}$$

$\boxed{\Sigma f_i = N}$ $$= \frac{\Sigma\, x_i \times f_i}{\Sigma f_i} = \frac{\Sigma x f}{N}$$ where $i = 1, 2, \cdots n$ ie $x_1, x_2, \cdots x_n$ like

Example 1: An Air India flight carried passengers from Delhi to Gauhati in a particular week like ~~this~~ the following. Find the average of daily passengers in that week.

Monday	Tuesday	Wednesday	Thurs	Fri	Sat.	Sunday
255	173	200	150	276	195	213

Solution: The mean is $\bar{x} = \frac{\Sigma x}{n}$

here $n = 7$ (7 days)

$\Sigma x = 255 + 173 + 200 + 150 + 276 + 195 + 213$

$= 1462$

$\therefore$ Average daily passengers $= \frac{1462}{7}$

$= 208.9$

$= 209$ (approximate)

209 Ans.

Example 2 The following is a list of income earned by daily wage earners. Find their average income.

daily income	200	500	900	1100	1300	Total
No. of worker	2	1	4	2	1	10

Soln: here $\sum_i f_i \,(= N) = 10$

$\Sigma x_i f_i = 200\times2 + 500\times1 + 900\times4 + 1100\times2 + 1300\times1$

$= 8000$

$\therefore \quad \bar{x} = \frac{8000}{10} = 800$. Ans: Rs 800

Note In calculating mean, we must assign the <u>unit</u> in the answer.

Other short cut Methods of finding A.M.

(i) $\bar{x} = A + \frac{\Sigma f_i d_i}{N}$ where A = Assumed Mean

$d = x_i - A$

$N = \sum_i f_i$

(ii) $\bar{x} = A + \frac{\Sigma f_i d_i'}{N} \times C$ where A = Assumed Mean

$d_i' = \frac{d_i}{\text{Common factor}}$

C = Common factor.

The above methods are known as Step deviation method.

Properties of A.M.

(A) The algebraic sum of the deviations of the values from their A.M. is zero.

Here deviations mean differences.

So, if $x_1, x_2, \ldots\ldots, x_n$ are the values and $\bar{x}$ is their A.M.

Then $(x_1-\bar{x}) + (x_2-\bar{x}) + (x_3-\bar{x}) + \cdots\cdots + (x_n-\bar{x}) = 0$

or. $\boxed{\sum_{i=1}^{n} (x_i - \bar{x}) = 0}$

Proof: $(x_1-\bar{x}) + (x_2-\bar{x}) + (x_3-\bar{x}) + \cdots\cdots + (x_n-\bar{x})$

$= (x_1 + x_2 + \cdots\cdots + x_n) - n\bar{x}$ [Here total no. of values is n]

$= \Sigma x_i - n\bar{x} = \frac{\Sigma x_i}{n} - \bar{x} = \bar{x} - \bar{x} = 0.$ Proof

Like that if x_1 has frequency f_1, x_2 has frequency f_2, ... x_n has frequency f_n,

Then $\boxed{\sum_{i=1}^{n} f_i (x_i - \bar{x}) = 0}$

(B) Arithmetic mean depends on both scale and origin.

? Find out A.M. from Grouped Data.

From Grouped data, we choose step deviation method to find weighted Arithmetic Mean.

Example Calculate mean.

$C.I$	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
f_i	0	4	10	22	30	35	10	7	1

Solⁿ:

C.I	x	f	$d = x - A$	$d' = \frac{d}{c}$	fd'
0-10	$\frac{0+10}{2} = 5$	0	-40	-4	0
10-20	$\frac{10+20}{2} = 15$	4	-30	-3	-12
20-30	25	10	-20	-2	-20
30-40	35	22	-10	-1	-22
40-50	$\boxed{45}$	30	0	0	0
50-60	55	35	10	1	35
60-70	65	10	20	2	20
70-80	75	7	30	3	21
80-90	85	1	40	4	4
		120			26

Take $A = 45$

Now, $A.M\ (\bar{x}) = A + \frac{\Sigma fd'}{\Sigma f} \times c$ | $\Sigma f = 120$, $c = 10$

$= 45 + \frac{26}{120} \times 10$

$= 45 + \frac{26}{12}$

$= 45 + 2.166$

$= 47.17$ (app.) Ans

We have to assign unit in the answer if unit is given in the question.

? For unequal class intervals or open end class intervals, same way we have to follow.

Like 1 — 3, 3 − 5, 5 — 10, 10 — 15, 15 − 25 etc

and below 5, 5 − 10, 10 − 15, 15 − 20, above 20.

? Combined Arithmetic mean

If there are two groups and A.M.s' of each group are given with the number of values in each group. Then their combined A.M is given by

$$\bar{x}_{12} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$\bar{x}_1$ = A.M. of 1st gp.
n_1 = No. of values in 1st gp.
$\bar{x}_2$ = A.M. of 2nd gp
n_2 = No. of values in 2nd gp.

2. Geometric Mean (G.M).

Definition : If $x_1, x_2, \ldots, x_n$ are n positive values, then their Geometric Mean is defined as

$$G = \sqrt[n]{x_1 . x_2 . \ldots x_n} \text{ or } (x_1 \times x_2 \times \ldots \times x_n)^{1/n}$$

If we take logarithim, then

$$G = \text{antilog}\left(\frac{1}{n} \Sigma \log x\right)$$

Simple examples of G.M

find G.M of 2, 4 and 8

Ans :
$$G = (2 \times 4 \times 8)^{1/3}$$
$$= (2 \times 2^2 \times 2^3)^{1/3}$$
$$= (2^3 \times 2^3)^{1/3}$$
$$= 2 \times 2$$
$$= 4 \text{ Ans}$$

Weighted Geometric Mean

If $f_1, f_2, f_3, \ldots, f_n$ are the respective frequencies of n variables $x_1, x_2, \ldots, x_n$, then their Geometric Mean is defined by

$$G = \left(x_1^{f_1} \times x_2^{f_2} \times \ldots \times x_n^{f_n}\right)^{\frac{1}{N}}$$

where $N = f_1 + f_2 + \cdots + f_n = \Sigma f$

$$\log G = \frac{1}{N}(f_1 \log x_1 + f_2 \log x_2 + \cdots + f_n \log x_n)$$

$$= \frac{1}{N} \Sigma f \log x$$

$$\therefore G = \text{Antilog}\left[\frac{1}{N} \Sigma f \log x\right].$$

Use of Geometric Mean

G.M. is used in the construction of Index numbers.

(3) Harmonic Mean (H.M.)

Definition: Harmonic Mean (H) of n observations (or values) $x_1, x_2, \ldots, x_n$ is defined as

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}} = \frac{n}{\Sigma \frac{1}{x}}.$$

If x_1 has frequency f_1, x_2 has frequency f_2, x_3 has frequency f_3, ... x_n has frequency f_n then their weighted H.M is defined as

$$H = \frac{N}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \cdots + \frac{f_n}{x_n}}$$

$$= \frac{N}{\Sigma \frac{f}{x}} \quad \text{where } N = f_1 + f_2 + \cdots + f_n.$$

Harmonic Mean is not commonly used. But it is useful in averaging speed when distance covered are equal.

Relationship between A.M., G.M. and H.M

(1) $A.M. \geq G.M \geq H.M.$

(2) For a pair of observations only

$$(G.M.)^2 = A.M \times H.M.$$

——— x ———