LINEAR INEQALITIES :

In mathematics a **linear inequality** is an inequality which involves a linear function. A linear inequality contains one of the symbols of inequality:[1]. It shows the data which is not equal in graph form.

- < less than
- > greater than
- ≤ less than or equal to
- ≥ greater than or equal to
- ≠ not equal to
- = equal to

A linear inequality looks exactly like a linear equation, with the inequality sign replacing the equality sign.

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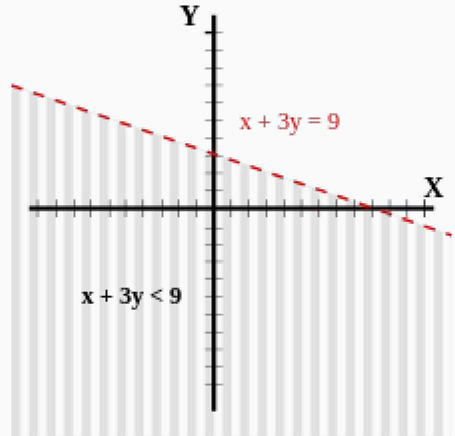

Graph of linear inequality:
x + 3y < 9

Two-dimensional linear inequalities are expressions in two variables of the form:

where the inequalities may either be strict or not. The solution set of such an inequality can be graphically represented by a half-plane (all the points on one "side" of a fixed line) in the Euclidean plane.

The line that determines the half-planes ($ax + by = c$) is not included in the solution set when the inequality is strict. A simple procedure to determine which half-plane is in the solution set is to calculate the value of $ax + by$ at a point (x_0, y_0) which is not on the line and observe whether or not the inequality is satisfied.

For example, to draw the solution set of $x + 3y < 9$,

one first draws the line with equation $x + 3y = 9$ as a dotted line, to indicate that the line is not included in the solution set since the inequality is strict. Then, pick a convenient point not on the line, such as (0,0). Since 0 + 3(0) = 0 < 9, this point is in the solution set, so the half-plane containing this point (the half-plane "below" the line) is the solution set of this linear inequality.

Linear inequalities in general dimensions:

In $\mathbf{R}^n$ linear inequalities are the expressions that may be written in the form

$$f(x) < b \text{ or } f(x \le b)$$

where f is a linear form (also called a *linear functional*),

$\bar{x} = (x_1,\ x_{2,}\ \ x_{3\,,,,} \ldots \ldots . x_n\ \ )$

b is the constant real number and this may be written as

$a_1x_{1+} a_2x_{2+} a_3x_{3+} \quad \ldots\ldots\ldots\ldots\ldots < b$

Or

$a_1x_{1+} a_2x_{2+} a_3x_{3+} \quad \ldots\ldots\ldots\ldots\ldots \leq b$

$x_1, x_2, x_3 \ldots\ldots x_n$ *are called unknown and* $a_1, a_2, \ldots\ldots a_n$ coefficients.

This may be written as $g(x) < 0 \ \ or\ g(x) \leq 0)$

Where g is a affine function. Then

$a_0 + a_1x_1 + a_2x_2 + \cdots \ldots\ldots\ldots . a_nx_n < 0 \ \ or \quad a_0 + a_1x_1 + a_2x_2 + \cdots \ldots\ldots\ldots . a_nx_n \leq 0$

Systems of linear inequalities:

A system of linear inequalities is a set of linear inequalities in the same variables:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots \ldots\ldots\ldots .. + a_{1n}x_n < b$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 \ldots . a_{2n}x_n < b_2$$

$$\ldots\ldots \quad \ldots\ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots \quad \ldots\ldots \quad \ldots\ldots\ldots$$

$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + a_{m4}x_4 \ldots\ldots\ldots .. a_{mn}x_n$$

Where $x_1, x_2, x_3 \ldots\ldots . x_n$ *are unknowns,* $a_{11}, a_{12}, a_{13} \ldots\ldots\ldots .. a_{mn}$ *are the coefficients of the system and*

$b_1, b_2, b_3, b_4, \ldots\ldots\ldots\ldots .. b_m$

are the constant terms. This can be written as the matrix inequality –

$Ax \leq b$ *where a is an* $m \times n$ *matrix* (m rows and n columns)

x *is an* $n \times 1$ *column vector of variable* , b *is an* $m \times 1$ *column vector of constant.*

Applications

Polyhedra

The set of solutions of a real linear inequality constitutes a half-space of the 'n'-dimensional real space, one of the two defined by the corresponding linear equation.

The set of solutions of a system of linear inequalities corresponds to the intersection of the half-spaces defined by individual inequalities. It is a convex set, since the half-spaces are convex sets, and the intersection of a set of convex sets is also convex. In the non-degenerate cases this convex set is a convex polyhedron (possibly unbounded, e.g., a half-space, a slab between two parallel half-spaces or a polyhedral cone). It may also be empty or a convex polyhedron of lower dimension confined to an affine subspace of the n-dimensional space $\mathbf{R}^n$.

Linear programming

Main article: Linear programming

A linear programming problem seeks to optimize (find a maximum or minimum value) a function (called the objective function) subject to a number of constraints on the variables which, in general, are linear inequalities.[6] The list of constraints is a system of linear inequalities.

Linear programming (**LP**, also called **linear optimization**) is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. Linear programming is a special case of mathematical programming (also known as mathematical optimization).

More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints. Its feasible region is a convex polytope, which is a set defined as the intersection of finitely many half spaces, each of which is defined by a linear inequality. Its objective function is a real-valued affine (linear) function defined on this polyhedron. A linear programming algorithm finds a point in the polytope where this function has the smallest (or largest) value if such a point exists.

Linear programs are problems that can be expressed in canonical form as

$$Maximize\ c^T x\ \ ; Subject\ to\ \ Ax \le b\ \ ; and\ x \ge 0$$

where **x** represents the vector of variables (to be determined), **c** and **b** are vectors of (known) coefficients, A is a (known) matrix of coefficients, and $(.)^T$ is the matrix transpose. The expression to be maximized or minimized is called the objective function ($\mathbf{c}^T\mathbf{x}$ in this case). The inequalities $A\mathbf{x} \le \mathbf{b}$ and $\mathbf{x} \ge \mathbf{0}$ are

the constraints which specify a convex polytope over which the objective function is to be optimized. In this context, two vectors are comparable when they have the same dimensions. If every entry in the first is less-than or equal-to the corresponding entry in the second, then it can be said that the first vector is less-than or equal-to the second vector.

Linear programming can be applied to various fields of study. It is widely used in mathematics, and to a lesser extent in business, economics, and for some engineering problems. Industries that use linear programming models include transportation, energy, telecommunications, and manufacturing. It has proven useful in modeling diverse types of problems in planning, routing, scheduling, assignment, and design.

Uses :

Linear programming is a widely used field of optimization for several reasons. Many practical problems in operations research can be expressed as linear programming problems.[3] Certain special cases of linear programming, such as *network flow* problems and *multicommodity flow* problems are considered important enough to have generated much research on specialized algorithms for their solution. A number of algorithms for other types of optimization problems work by solving LP problems as sub-problems. Historically, ideas from linear programming have inspired many of the central concepts of optimization theory, such as *duality, decomposition,* and the importance of *convexity* and its generalizations. Likewise, linear programming was heavily used in the early formation of microeconomics and it is currently utilized in company management, such as planning, production, transportation, technology and other issues. Although the modern management issues are ever-changing, most companies would like to maximize profits and minimize costs with limited resources. Therefore, many issues can be characterized as linear programming problems.

Standard Form :

Standard form is the usual and most intuitive form of describing a linear programming problem. It consists of the following three parts:

Standard form is the usual and most intuitive form of describing a linear programming problem. It consists of the following three parts:

- A linear function to be maximized

$$f(x_1, x_2) = c_1x_1 + c_2x_2$$

- Problem constraints of the following form

$$a_{11}x_1 + a_{12}x_2 \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 \leq b_2$$

$$a_{31}x_1 + a_{32}x_2 \leq b_3$$

- Non-negative variables

$$x_1 \geq 0 \quad ; x_2 \geq 0$$

The problem is usually expressed in matrix form and then becomes:
$\max\{c^T x\}\ Ax \leq b \wedge x \geq 0$

Example

Suppose that a farmer has a piece of farm land, say L km^2, to be planted with either wheat or barley or some combination of the two. The farmer has a limited amount of fertilizer, F kilograms, and pesticide, P kilograms. Every square kilometer of wheat requires F_1 kilograms of fertilizer and P_1 kilograms of pesticide, while every square kilometer of barley requires F_2 kilograms of fertilizer and P_2 kilograms of pesticide. Let S_1 be the selling price of wheat per square kilometer, and S_2 be the selling price of barley. If we denote the area of land planted with wheat and barley by x_1 and x_2 respectively, then profit can be maximized by choosing optimal values for x_1 and x_2. This problem can be expressed with the following linear programming problem in the standard form:

$$Maximize\ S_1 x_1 + S_2 x_2\ (maximize\ the\ reveue - revenue\ is\ the\ objective\ function)$$

$$Subject\ to:\ x_1 + x_2 < L\ (Limit\ on\ total\ area)$$

$$F_1 x_1 + F_2 x_2 \leq F\ (Limit\ to\ fertilizer)$$

$$P_1 x_1 + P_2 x_2 \leq P\ (\ Limit\ to\ pesticide)$$

$$x_1 > 0,\quad x_2 > 0\ ;(Can't\ plant\ a\ negative\ area)$$

In matrix form this becomes we can write as

$$maximize:\ [S_1 \quad S_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{Subject to}:\ \begin{bmatrix} 1 & 1 \\ F_1 & F_2 \\ P_1 & P_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} L \\ F \\ P \end{bmatrix};\ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Augmented Form :

Linear programming problems can be converted into an *augmented form* in order to apply the common form of the simplex algorithm. This form introduces non-negative slack variables to replace inequalities with equalities in the constraints. The problems can then be written in the following block matrix form:

Maximize z

$$\begin{bmatrix} 1 & -c^T & 0 \\ 0 & A & I \end{bmatrix} \begin{bmatrix} z \\ x \\ s \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix}$$

$x \geq 0,\ s \geq 0$ *where is the nearlyintroduced slack variable, x is the derivation variable and z is the variable to be maximized.*

Example:

The example above is converted into the following augmented form:

$$Maximize:\ S_1x_1 + S_2x_2 \ \ _{objective\ function}$$

$$Subject\ to:\ x_1 + x_2 + x_3 = L\ (augmented\ constraint)$$

$$F_1x_1 + F_2x_2 + x_4 = F\ \ (augmented\ constraint)$$

$$P_1x_1 + P_2x_2 + x_5 = P\ \ (augmented\ constraint)$$

Where x_3, x_4, x_5 are the nonnegative slack variables. representing in this example the unused area, the amount of unused fertilizer, and the amount of unused pesticide.

In matrix form this becomes:

Maximize z=

(1 $-S_1$ $-S_2$ 0 0 0

0 1 1 1 0 0

0 F_1 F_2 0 1 0

0 P_1 P_2 0 0 1)

(z

X_1

X_2

X_3

X_4

X_5)

= (0

L

F

P)   ;

(x_1

x_2

x_3

x_4

x_5) ≥ 0

****