H S II Year Commerce - 2020
SUBJECT: C M S
R G Baruah College
Date: 9th August to 10th August,

Chapter II [Simple Interest & Compound Interest]

Simple Interest :

Simple interest is interest calculated on the principal portion of a loan or the original contrib. ution to a savings account. The formula is

S.I = Prt /100

Compound Interest : Compound interest (or compounding interest) is interest calculated on the initial principal, which also includes all of the accumulated interest from previous periods on a deposit or loan.

Periodic compounding[edit]

The total accumulated value, including the principal sum plus compounded interest , is given by the formula:[2][3]

$$P' = P\left(1+\frac{r}{n}\right)^{nt}$$

Where

P is the original principal sum
P' is the new principal sum
r is the nominal annual interest rate
n is the compounding frequency
t is the overall length of time the interest is applied (expressed using the same time units as r, usually years).

The total compound interest generated is the final value minus the initial principal:[4]

$$I = P\left(1 + \frac{r}{n}\right)^{nt} - P$$

The total compound interest generated is the final value minus the initial principal:[4]

The **Simple Interest Formula** is given by

Simple Interest = Principal × Interest Rate × Time

I = Prt

where

The **Principal (P)** is the amount of money deposited or borrowed.

The **Interest Rate (r)** is a percent of the principal earned or paid.

The **Time (t)** is the length of time the money is deposited or borrowed.

Exercise No 1:

Sarah deposits Rs. 4,000 at a bank at an interest rate of 4.5% per year. How much interest will she earn at the end of 3 years?

Solution:

Simple Interest = 4,000 × 4.5% × 3 = 540

She earns Rs. 540 at the end of 3 years.

Exercise No 2:

Wanda borrowed $3,000 from a bank at an interest rate of 12% per year for a 2-year period. How much interest does she have to pay the bank at the end of 2 years?

Solution :

Simple Interest = 3,000 × 12% × 2 = 720

She has to pay the bank $720 at the end of 2 years.

Exercise No 3:

Raymond bought a car for Rs. 40, 000. He took a Rs. 20,000 loan from a bank at an interest rate of 15% per year for a 3-year period. What is the total amount (interest and loan) that he would have to pay the bank at the end of 3 years?

Solution :

Simple Interest = 20,000 × 13% × 3 = 7,800

At the end of 3 years, he would have to pay

Rs.20,000 + Rs.7,800 = Rs. 27,800

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Tutorial on Simple Interest

Examples:

1. Ian is investing Rs.4,000 for 2 years. The interest rate is 5.5%. How much interest will Ian earn after 2 years?

2. Doug made a 3 year investment. The interest rate was 4.5%. After 3 years, he earned Rs.675 in interest. How much was his original investment?

3. Kim got a loan of Rs.4700 to buy a used car. The interest rate is 7.5%. She paid Rs. 1057.50 in interest. How many years did it take her to pay off her loan?

TRY AT HOME

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1. Find the Principal when.....

(a) S.I. = Rs. 192 Rate = 6% per annum Time = 4 years

(b) S.I = Rs. 20 Rate = 2% per annum Time = 20 month

2. Find the Rate when.....

(a) Principal = Rs. 350 Time = $2^1/_2$ years S.I. = Rs. 140

(b) Principal = Rs. 9600 Time = 3 months S.I = Rs. 72

ANSWERS: 1. (a) Rs.800 (b) Rs.600

2. (a) 16% (b) 3%

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You invest 100 (principal sum) and the return you expect is 10% (interest rate).

End of year 1 your investment will then have made you 10 as 10% of 100 is 10. So end of year you principal sum plus the return of 10 is then 110.

If you decide that you want to invest again in year 2 and you don't take out the 10% interest you just earned but instead you reinvest it then your investment in year 2 would be 110 and the interest is again 10%.

At the end of year 2 you would now then have made 10% on 110 and not on 100. In other words your return at the end of year 2 is 11 (10% of 110).

Now compare the two: 10 and 11.

What has happened here is the effect of compounded interest!

You have compounded the interest from year 1 which was 10 to 11 in year two.

Not only have you now increased your principal amount from 100 to 121 (100+10+11) but have now made the interest received also work for you as they say in finance, i.e. you have reinvested your returns and not taken them out.

It may seem like a small amount to receive 1 extra unit but next year it will increase and the next and the next like this:

100*10%=110

110*10%=121

121*10%=133.1

133.1*10%=146.14 etc...

If you would have invested only the principal amount each year you would only have 140 and not 146.41.

Imagine now if the investment is not 100 but instead 1 000 000! Compound interest is great when investing. But also remember when you are borrowing money, then compound interest goes aginst you as you will start paying interest on interest.

Please find our compound interest formulas below:

Annually compounded interest formula:

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^{n}$$

Half-yearly compounded interest formula:

$$\text{Amount} = P\left[1 + \frac{(R/2)}{100}\right]^{2n}$$

Quarterly compounded interest formula:

$$\text{Amount} = P\left[1 + \frac{(R/4)}{100}\right]^{4n}$$

Compounded interest formula with varying interest rates:

Compound Interest Exercises [Hints : $= Indian Rs.)

1. A man invests $3000 for 5 years at 4.2% compounded monthly.

A. How much will be in the account at the end of the term?

B. How much interest will be earned?

C. What is the annual percentage yield?

D. Prepare a table that shows the growth of the investment.

E. Create a line graph illustrating the growth of the investment.

2. A woman invests $5,000 in a 18-month CD (Certificate of Deposit) paying 5.2% compounded quarterly.

A. What is the future value of the CD?

B. How much interest will be earned?

C. What is the annual percentage yield?

D. Prepare a table that shows the growth of the investment.

E. Create a line graph illustrating the growth of the investment.

3. A $500 savings bond pays 6.5% compounded semiannually and matures in 7.5 years.

A. What is the purchase price?

B. How much interest will be earned?

C. What is the annual percentage yield?

D. Prepare a table that shows the growth of the investment.

E. Create a line graph illustrating the growth of the investment.

4. A couple decides to invest some money in an account for use during retirement. Their goal is to have $100,000 in 40 years. The account pays 5.5% compounded monthly.

A. How much do they need to invest to meet their goal?

B. How much interest will they earn?

C. What is the annual percentage yield?

D. Prepare a table that shows the growth of the investment.

E. Create a line graph illustrating the growth of the investment.

ANSWERS:

These are the solutions to the compound interest exercises. You are strongly advised to work out your own solutions before you look at these.

1. A man invests $3000 for 5 years at 4.2% compounded monthly.

A. How much will be in the account at the end of the term?

The formula yields a future value of $3,699.68 but the table results in a value of $3,699.67.

B. How much interest will be earned?

The formula yields a total interest of $699.68 but the table results in $699.67 of total interest.

C. What is the annual percentage yield?

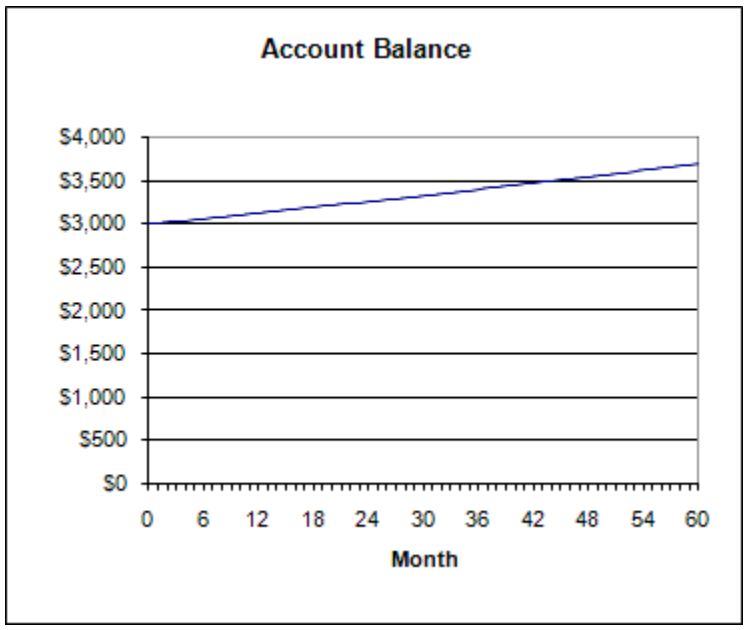
APY = 4.282% (rounded to 3 decimal places)

Calculator solution for A, B and C

D. Prepare a table that shows the growth of the investment.

Period	Initial Balance	Periodic Interest	Final Balance
0			3,000.00
1	3,000.00	10.50	3,010.50
2	3,010.50	10.54	3,021.04
⋮			
58	3,661.10	12.81	3,673.91
59	3,673.91	12.86	3,686.77
60	3,686.77	12.90	3,699.67

E. Create a line graph illustrating the growth of the investment.



2. A woman invests $5,000 in a 18-month CD (Certificate of Deposit) paying 5.2% compounded quarterly.

A. What is the future value of the CD?

Based on the formula, the future value is $5,402.90. The table results in a future value of $5,402.91

B. How much interest will be earned?

Based on the formula, the total interest is $402.90. The table results in a total interest of $402.91.

C. What is the annual percentage yield?

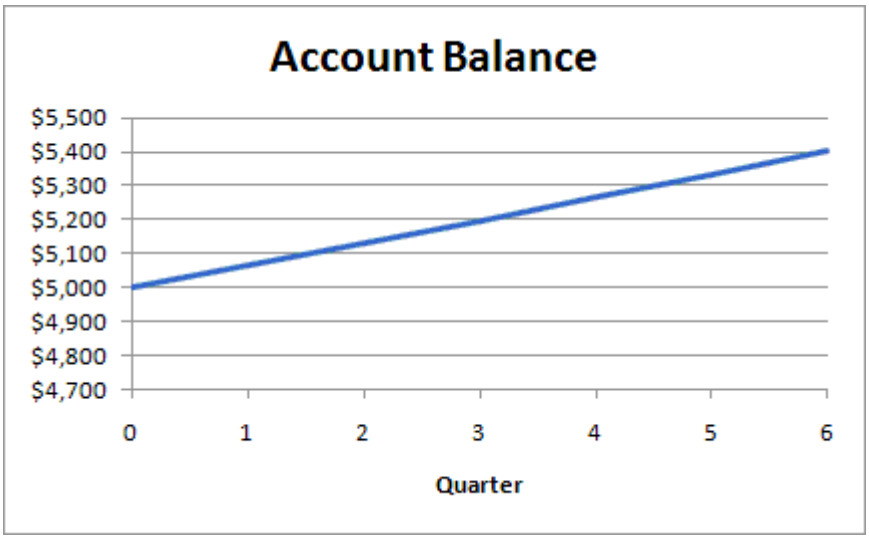
APY = 5.302% (rounded to 3 decimal places)

Calculator solution for A, B and C

D. Prepare a table that shows the growth of the investment. (See below.) Notice that the actual interest and future value are one cent more than the results given by the formulas.

Period	Initial Balance	Periodic Interest	Final Balance
0			5,000.00
1	5,000.00	65.00	5,065.00
2	5,065.00	65.85	5,130.85
3	5,130.85	66.70	5,197.55
4	5,197.55	67.57	5,265.12
5	5,265.12	68.45	5,333.57
6	5,333.57	69.34	5,402.91

F. Create a line graph illustrating the growth of the investment.



3. A $500 savings bond pays 6.5% compounded semiannually and matures in 7.5 years.

A. What is the purchase price? [**Hints** $ = Indian Rs,]

The purchase price is $309.47.

B. How much interest will be earned?

The total interest will be $190.53.

C. What is the annual percentage yield?

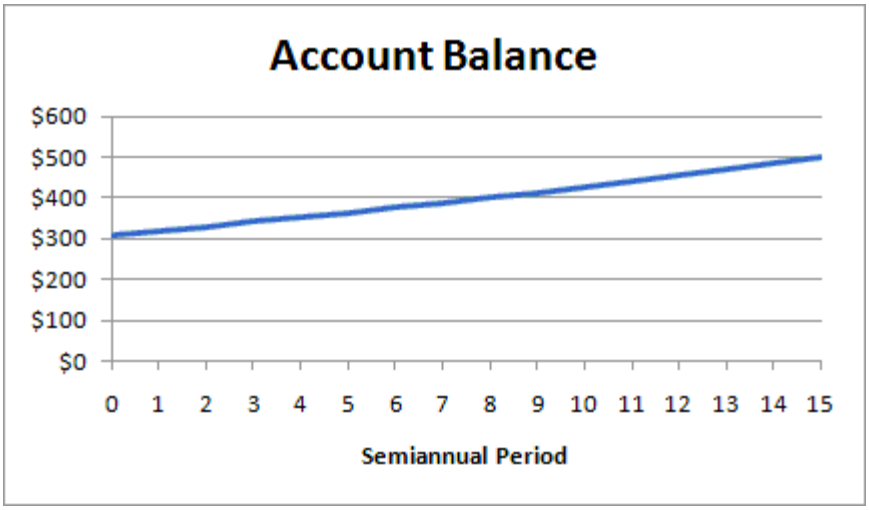
APY = 6.606% (rounded to 3 decimal places)

Calculator solution for A, B and C

D. Prepare a table that shows the growth of the investment. Notice that, once more, the table values and the formula values differ by a penny.

Period	Initial Balance	Periodic Interest	Final Balance
0			309.47
1	309.47	10.06	319.53
2	319.53	10.38	329.91
3	329.91	10.72	340.63
4	340.63	11.07	351.70
5	351.70	11.43	363.13
6	363.13	11.80	374.93
7	374.93	12.19	387.12
8	387.12	12.58	399.70
9	399.70	12.99	412.69
10	412.69	13.41	426.10
11	426.10	13.85	439.95
12	439.95	14.30	454.25
13	454.25	14.76	469.01
14	469.01	15.24	484.25
15	484.25	15.74	499.99

E. Create a line graph illustrating the growth of the investment.



4. A couple decides to invest some money in an account for use during retirement. Their goal is to have $100,000 in 40 years. The account pays 5.5% compounded monthly.

A. How much do they need to invest to meet their goal?

They will need to invest $11,136.15.

B. How much interest will they earn?

Over the 40-year term of the investment, they will earn a total of $88,863.85 in interest.

C. What is the annual percentage yield?

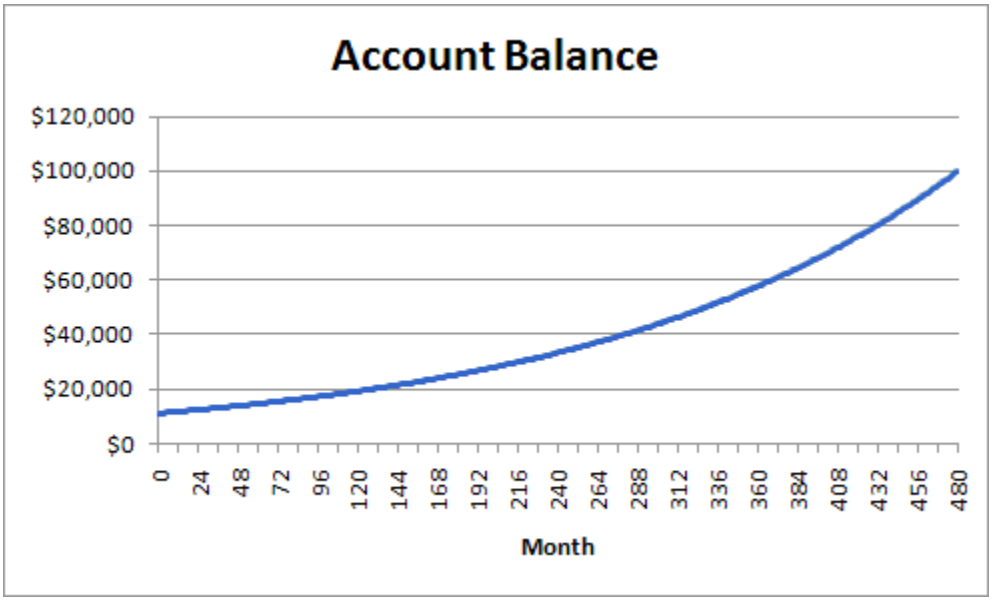
APY = 5.641% (rounded to 3 decimal places)

Calculator solution for A, B and C

D. Prepare a table that shows the growth of the investment.

Period	Initial Balance	Periodic Interest	Final Balance
0			11,136.15
1	11,136.15	51.04	11,187.19
2	11,187.19	51.27	11,238.46
⋮			
478	98,637.51	452.09	99,089.60
479	99,089.60	454.16	99,543.76
480	99,543.76	456.24	100,000.00

E. Create a line graph illustrating the growth of the investment.



See the whole Excel workbook.

Compound Interest Formula :

The mathematical formula for calculating compound interest depends on several factors. These factors include the amount of money deposited called the principal, the annual interest rate (in decimal form), the number of times the money is compounded per year, and the number of years the money is left in the bank. These factors lead to the formula

FV = future value of the deposit

P = principal or amount of money deposited

r = annual interest rate (in decimal form)

n = number of times compounded per year

t = time in years.

$$FV = P\left(1+\frac{r}{n}\right)^{nt}$$

Example 1:

If you deposit Rs.4000 into an account paying 6% annual interest compounded quarterly, how much money will be in the account after 5 years? [Hints: $ = Indian Rs.]

Plug in the giving information, P =Rs. 4000, r = 0.06, n = 4, and t = 5.

Use the order or operations to simplify the problem.

If the problem has decimals, keep as many decimals as possible until the final step.

$$FV = 4000\left(1+\frac{0.06}{4}\right)^{4(5)}$$

FV =4000(1.015)^{20}

FV 4000(1.346855007)

FV = 5387.42 ; FV = 5387.42 Round your final answer to two decimals places.

After 5 years there will be $5387.42 in the account.

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CHAPTER III - Logarithm

Rules or Laws of Logarithms

In this lesson, we will discuss the common rules of logarithms, also known as the "log rules". These seven (7) log rules are useful in expanding logarithms, condensing logarithms, and solving logarithmic equations. In addition, since the inverse of a logarithmic function is an exponential function, I would also recommend that you go over and master the exponent rules. Believe me, they always go hand in hand.

Rules of Logarithms

Rule 1: $\log_b(M \cdot N) = \log_b M + \log_b N$

Rule 2: $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

Rule 3: $\log_b\left(M^k\right) = k \cdot \log_b M$

Rule 4: $\log_b(1) = 0$

Rule 5: $\log_b(b) = 1$

Rule 6: $\log_b\left(b^k\right) = k$

Rule 7: $b^{\log_b(k)} = k$

Where:

$b > 0$ but $b \neq 1$, and M, N, and k are real numbers but M and N must be positive!

Descriptions of Logarithm Rules

Rule 1: Product Rule

$$\log_b (M \cdot N) = \log_b M + \log_b N$$

The logarithm of the product is the sum of the logarithms of the factors.

Rule 2: Quotient Rule

$$\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$$

The logarithm of the ratio of two quantities is the logarithm of the numerator minus the logarithm of the denominator.

Rule 3: Power Rule

$$\log_b \left(M^k\right) = k \cdot \log_b M$$

The logarithm of an exponential number is the exponent times the logarithm of the base.

Rule 4: Zero Rule

$$\log_b (1) = 0$$

The logarithm of 11 such that b > 0b>0 but b≠1 equals zero.

Rule 5: Identity Rule

$$\log_b (b) = 1$$

The logarithm of the argument (inside the parenthesis) wherein the argument is the same as the base is equal to 11. Since the argument is equal to the base, b*b* must be greater than 00 but it cannot equal to 11.

Rule 6: Log of Exponent Rule (Logarithm of a Base to a Power Rule)

$$\log_b\left(b^k\right)=k$$

The logarithm of an exponential number where its base is the same as the base of the log is equal to the exponent.

Rule 7: Exponent of Log Rule (A Base to a Logarithmic Power Rule)

$$b^{\log_b(k)}=k$$

Raising the logarithm of a number to its base is equal to the number.

Example 1:
Evaluate the expression below using Log Rules.

$Log_2 8 + log_2 4$

$$\log_2 8+\log_2 4=\log_2 2^3+\log_2 2^2$$
$$=3\log_2 2+2\log_2 2$$
$$=3(1)+2(1)$$
$$=3+2$$
$$\log_2 8+\log_2 4=5$$

ANSWER: 5

Example 2: Evaluate the expression below using Log Rules.

$\log_3 162 - \log_3 2$

$$\log_3 162 - \log_3 2 = \log_3 \left(\frac{162}{2}\right)$$
$$= \log_3 (81)$$
$$= \log_3 3^4$$
$$= 4 \log_3 3$$
$$= 4(1)$$
$$\log_3 162 - \log_3 2 = 4$$

Answer : 4

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Example 3: Evaluate the expression below.

$$\log_5 500 - 2\log_5 2 + \log_4 32 + \log_4 8$$

$$\log_5 500 - 2\log_5 2 + \log_4 32 + \log_4 8 = \log_5 500 - \log_5 2^2 + \log_4 32 + l$$
$$= \log_5 500 - \log_5 4 + \log_4 32 + lo$$
$$= \log_5\left(\frac{500}{4}\right) + \log_4(32 \cdot 8)$$
$$= \log_5(125) + \log_4(256)$$
$$= \log_5\left(5^3\right) + \log_4\left(4^4\right)$$
$$= 3 \cdot \log_5 5 + 4 \cdot \log_4(4)$$
$$= 3(1) + 4(1)$$
$$= 3 + 4$$
$$= 7$$

ANSWER: 7

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Example 4: Expand the logarithmic expression below.

$$\log_3\left(27x^2y^5\right) = \log_3(27) + \log_3\left(x^2\right) + \log_3\left(y^5\right)$$
$$= \log_3\left(3^3\right) + \log_3\left(x^2\right) + \log_3\left(y^5\right)$$
$$= 3\log_3(3) + 2\log_3(x) + 5\log_3(y)$$
$$= 3(1) + 2\log_3(x) + 5\log_3(y)$$
$$\log_3\left(27x^2y^5\right) = 3 + 2\log_3(x) + 5\log_3(y)$$

ANSWER: $[3+2\text{Log}_3(x) +5\ \text{Log}_3(y)]$

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